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type of mutation beyond cavil or doubt. In the second place, there is a considerable series of forms which depend upon duplication of one or more or even all of the chromosomes to the extent of tetraploidy in some forms. The particular type of behavior displayed by such forms appears to depend upon changes in the proportions of the elements within the reaction systems, rather than upon actual changes in germinal substance. In the third place, there is a complicated group of phenomena which appear to be best considered as due to complex segregation of a type analogous to that displayed in wide crosses. In contrast to the simple and definite behavior of factor mutants, the forms resulting from this segregation are often distinctly different throughout from the forms from which they arose, and when tested with them, they exhibit a complicated but orderly type of hereditary behavior. There are two facts which stand out prominently with respect to this behavior—first the mutations affect the total ontogenetic development of the individual and second they tend to recur in relatively constant ratios in certain races. The definite ratio relations in the production of 'mutant' forms, the peculiar but orderly behavior of the hybridization phenomena, and the universal occurrence of partial sterility make together a series of facts which seem at least as consistently explainable on the basis of substratum hybridity as on assumptions of general germinal change. If the conceptions applied above to the behavior of species hybrids be extended in a somewhat modified form to the *Oenothera* phenomena, the occurrence of the 'mutants' and their subsequent behavior in hybridization admit of logical arrangement and interpretation without any necessity for assumptions of extensive germinal changes.

The experimental data cited above were obtained from cultures made possible by a portion of the Adams' Fund allotted to the Department of Botany by the Department of Agriculture of the University of California. A more detailed statement of the general position here outlined has been prepared and will appear in the near future.

POINT SETS AND ALLIED CREMONA GROUPS (PART II)

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In Part I of this account[1] the ordered set P_n^k of n discrete points in a projective space S_k was studied with particular reference to its invariants, its association with a set Q_n^{n-k-2}, and its mapping upon a space $\Sigma_{k(n-k-2)}$. In this space Σ there was induced by permutation of the points of P_n^k a Cremona group $G_{n!}$ which for $n = 5, 6$ led to new versions of the solutions of the quintic and sextic equations.

In Part II the novel idea of the *congruence* of point sets is introduced. This notion can be defined at once for a space S_2. Two mutually ordered point sets P_n^2 and $P_n'^2$ are congruent under a ternary Cremona transformation C with $\rho \leqq n$ fundamental points (F = *points*) if ρ of the pairs comprise the F-points of C and C^{-1} and if the remaining $n-\rho$ pairs of the sets are pairs of ordinary corresponding points of C. Thus the number of types of congruence depends upon the number of types of C and the number of ways of ordering the two sets. There is a natural pairing of the ρ F-points of C with the ρ F-points of C^{-1} and this is utilized to state in quite compact form the conditions for congruence up to $n = 9$. These conditions imply when $\rho = n$ a construction for the two sets of F-points and when $\rho < n$ a construction for the transformation C. The importance of the notion of congruence is due to the fact that two sets congruent in some order to a third are congruent in some order to each other. Thus if all the sets $P_n'^2$ congruent in some order to a given set P_n^2 be mapped upon points P' of $\Sigma_{2(n-4)}$ they form a conjugate set of points under the operations of a Cremona group $G_{n,2}$ in $\Sigma_{2(n-4)}$ which contains the $G_{n!}$ of Part I as a subgroup.

This definition of congruence cannot be extended immediately to sets in S_k. Let us first define a *regular* Cremona transformation C in S_k to be one which can be generated as a product of projectivities and of inversions of the variables. The regular transformations constitute a *regular Cremona group in* S_k. They are determined by their F-points precisely as in S_2. It is now possible to define as above congruence of sets P_n and $P_n'^k$ under regular Cremona transformation in S_k and the group $G_{n,k}$ in $\Sigma_{k(n-k-2)}$.

The effect of a regular transformation on spreads in S_k of order x_0 with multiple points of orders $x_1, \ldots x_n$ at the points of P_n^k is represented by a linear transformation on these variables with integral coefficients. In this way a group $g_{n,k}$ is derived which is isomorphic with $G_{n,k}$. This group reveals the striking analogy between the general transformation in S_2 and the regular transformation in S_k.

All sets $P_n'^k$ congruent to a set P_n^k upon an elliptic norm-curve in S_k can be projected upon the same curve. The transition from P_n^k to $P_n'^k$ is then effected by a linear transformation on the elliptic parameters $u_1, \ldots, u_n$ with rational coefficients. In this way a group $e_{n,k}$ is derived which also is isomorphic with $G_{n,k}$. The group is useful in determining the cases in which congruence implies projectivity.

The close relation of associated sets P_n and Q_n^{n-k-2} is again apparent in that the groups $G_{n,k}$ and $G_{n,\, n-k-2}$ are identical while the groups $g_{n,\, k}$ and $g_{n,\, n-k-2}$, as well as the groups $e_{n,\, k}$ and $e_{n,\, n-k-2}$, are linear transforms of each other.

The group $G_{n,\, k}$ is in general infinite and discontinuous. The only finite types are the $G_{6,\, 2}$, the $G_{7,\, 2} = G_{7,\, 3}$ and the $G_{8,\, 2} = G_{8,\, 4}$. These are identified with the well known groups of respectively the lines on a cubic surface, the bitangents of a quartic, and the tritangent planes of a sextic of genus 4 on a quadric cone. For these cases $G_{n,\, k}$ is in immediate algebraic relation to the corresponding geometric configuration. This advantage is used in the case of P_7^2 to determine the simplest system of irrational invariants of the point set which are invariants of the allied quartic as well. A similar method will be employed in Part III to handle the P_6^2 and the allied cubic surface. The first cases of infinite order, the $G_{9,\, 2} = G_{9,\, 5}$ and the $G_{8,\, 3}$ can be adequately discussed by means of the isomorphic group $e_{n,k}$ and their structure has been determined.

Some interesting by-products are obtained. By means of $g_{n,\, k}$ a determination of all types of regular Cremona transformations with a single symmetrical set of F-points is made. The new types thus found are a transformation in S_4 of order 49 with 8 F-points of order 30 and a transformation in S_{2k-1} of order $(2k^2 - 1)$ with $2\ (k + 1)$ F-points of order $2k\ (k - 1)$. Also the discussion of $e_{9,\, 2}$ leads to a determination of the infinite number of types of ternary Cremona transformations with 9 or fewer F-points in terms of 8 independent integers. Theorems such as the following:—*A pencil of plane cubic curves can be transformed by ternary Cremona transformation into only* 960 *projectively distinct pencils of cubics*—are proved for special sets P_9^2. Similar facts can be derived for P_8^3. Furthermore the general methods developed can be employed in the problem of determining the finite groups of regular transformations in S_k. For if a point P in $\Sigma_{k(n-k-2)}$ is fixed under a certain subgroup of $G_{n,k}$ the corresponding set P_n^k in S_k defines a regular Cremona group in S_k isomorphic with the given subgroup.

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[1] These PROCEEDINGS, **1**, 245 (1915) and *Trans. Amer. Math. Soc.*, **16**, 155 (1915). This investigation has been carried out under the auspices of the Carnegie Institution of Washington, D. C.